Traffic flow in a Manhattan-like urban system

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In this paper, a cellular automaton model of vehicular traffic in Manhattan-like urban system is proposed. In this model, the origin-destination trips and traffic lights have been considered. The system exhibits three different states, i.e., moving state, saturation state and global deadlock state. With a grid coarsening method, vehicle distribution in the moving state and the saturation state has been studied. Interesting structures (e.g., windmill-like one, T-shirt-like one, Y-like one) have been revealed. A metastability of the system is observed in the transition from saturation state to global deadlock state. The effect of advanced traveller information system (ATIS), the traffic light period, and the traffic light switch strategy have also been investigated.

I. INTRODUCTION

In modern society, the transportation of people and goods as well as information are becoming more and more frequent. As a result, in transportation and communication systems, traffic congestion has become one of the urgent issues to be tackled [1-3]. Recent researches in the field of network traffic of information packet indicate that the network capacity could be remarkably improved by optimizing the routing strategy [4-10]. Therefore, it is natural to expect that the efficiency of transportation network of vehicles could also be improved in the same way, in particular with the help of advanced traveller information system (ATIS).

Recently, Scellato et al have studied the vehicular flow in urban street network [11]. They have investigated a congestion aware routing strategy, which allows the vehicles to dynamically update the routes towards their destinations. It is shown that in real urban street network of various cities, a global traffic optimization could be achieved based on local agent decisions.

Nevertheless, we would like to point out that in the work of Scellato et al., the influence of traffic lights has not been taken into account. In their model, the vehicles coming from different adjacent streets compete for the same intersection. Since traffic lights play a very important role in urban traffic, their effect needs to be carefully investigated. Moreover, by using the routing strategy in Ref.[11], another deficiency is that there might be some vehicles which could never reach their destinations (or it takes extremely long time to reach the destinations) because they are always detouring around the destinations.

In this paper, we propose a model to study urban traffic considering the traffic lights. For simplicity, our model is implemented in a Manhattan-like urban system. In such system,

there are usually more than one shortest path between an origin and destination pair [19]. Therefore, also for simplicity, vehicles are only allowed to go along shortest paths in this paper. The routing strategy determines which shortest path to choose at the intersections. It is shown that three different states are observed in our model. With the help of ATIS and by choosing proper traffic light period, the transition to global deadlock state could happen at larger density.

This paper is organized as follow. The urban traffic model and routing strategy will be presented in section II. In section III, the simulation results will be discussed. Finally, we present the conclusion and outlook in section IV.

II. MODEL

In our model, urban city is presented as a Manhattan-like system as shown in Fig.1(a). We model the network of streets as a $N \times N$ square lattice. The spatial separation between any two successive intersections is assumed to have one lane for each direction, and each lane is divided into L cells. Vehicle driving is restricted to the right lane. On each lane, the cells are numbered $1, 2, \ldots, L$ from downstream to upstream. For simplicity, we assume that synchronous traffic lights with fixed period are set at each intersection. In each traffic phase, the traffic lights keep green for one ingoing street and red for other three ingoing streets for T time steps. Therefore, the traffic light period is $4T$ (the yellow light is not considered). When the green light is on, vehicles on the corresponding ingoing street could go straight ahead, turn left or right, or make a U-turn. In this way, the conflicts between turning vehicles and straight vehicles are avoided. Although our setup is simplified, it could capture the essence of traffic congestion in urban area, i.e., the spillover of queues from downstream intersection to upstream ones.

The state of the system is updated by applying the Nagel-Schreckenberg (NS) rules [12] to each vehicle in parallel, except the leading vehicles on each road. The NS rules are as follow:
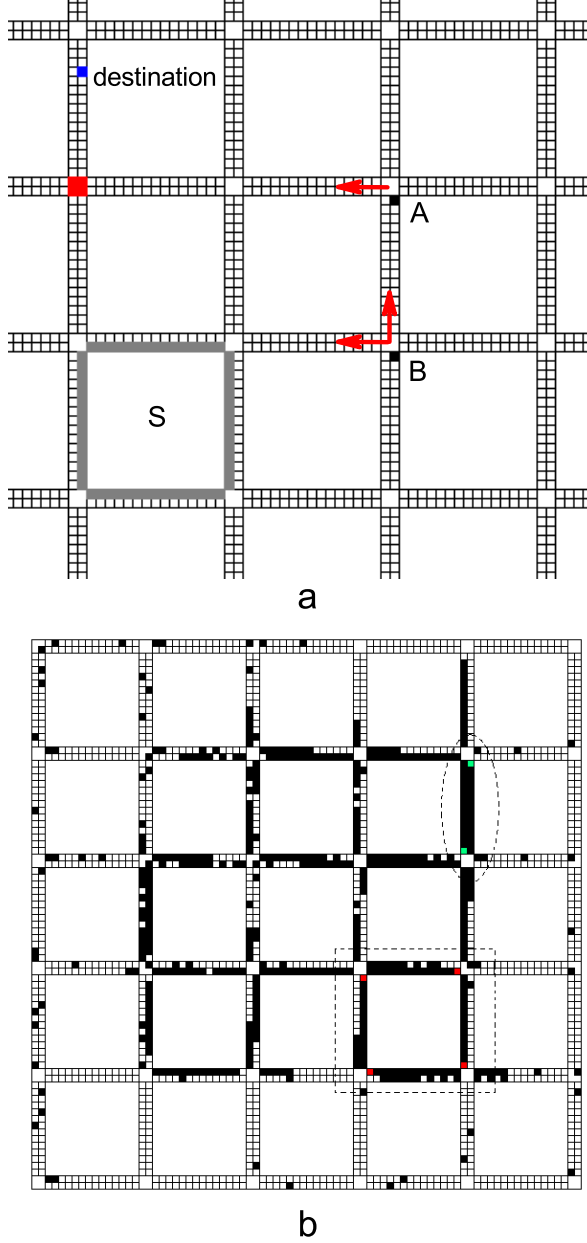
a

b

FIG. 1: (Color online) (a) Example of a Manhattan-like urban system. Each cell can either be empty or occupied by a vehicle. The black sites are occupied by vehicle A and B. The blue site is their destination. As right-hand side driving, the vehicles must go to the red intersection first. Currently, there are two directions for vehicle B to choose, but only one direction for vehicle A (shown by red arrows). The vehicle density of lattice S, is the average vehicle density of the four roads (grey roads) around S. (b) Snapshot of two local deadlocks in a small-sized system. The loop structure of the local deadlock is indicated by the dashed lines. The four leading vehicles (indicated by red) in the square loop wish to make right turn; the two leading vehicles (indicated by green) in the oval loop wish to make U-turn.

- Acceleration: $v_n \rightarrow \min(v_n + 1, v_{max})$;

- Deceleration: $v_n \rightarrow \min(v_n, d_n)$;

- Random brake: $v_n \rightarrow \max(v_n - 1, 0)$ with a braking probability p;

- Movement: $x_n \rightarrow x_n + v_n$;

where v_{max} is the maximum velocity of vehicles, x_n is the position of the nth vehicle on each road and $d_n = x_{n+1} - x_n - 1$ is the distance to the vehicle ahead.

In our model, once an outgoing street is in jam, the vehicles choosing that outgoing street are not allowed to enter the intersection to avoid hindering vehicles on other streets. We assume that an outgoing street is in jam once its last two cells are simultaneously occupied.

As a result, update rules of the leading vehicles are the same as NS rules except the definition of d. The details are listed below:

(1) Traffic light is green

- if the desired outgoing street is in jam or the intersection is occupied by other vehicle which needs additional time to drive into its desired outgoing street (if the yellow traffic light period is taken into account, the latter situation would hardly occur), d is the distance to the intersection;

- otherwise, d is the distance to the last vehicle of the desired outgoing street.

(2) Traffic light is red

- d is the distance to the intersection.

Each vehicle has its origin and destination. Initially, all vehicles randomly select their origin and destination. When a vehicle reaches its destination, a new destination is chosen randomly from the system beyond the current road. Thus the number of vehicles is conserved. As mentioned before, in our model vehicles are only allowed to go along shortest paths, but the drivers need to determine which outgoing street to run into at each intersection when it is necessary. As shown in Fig.1(a), when a vehicle reaches an intersection, two situations could appear: the vehicle either has two directions (vehicle B) or only one direction (vehicle A) to choose. We assume that without the help of ATIS, the driver will choose a direction randomly in the former situation.

Nowadays, ATIS could provide real-time information to the drivers. We suppose that at each intersection, the ATIS provides mean velocity on each outgoing street, and the drivers will choose the direction with larger value of mean velocity [13]. We have also tested the usage of density information and congestion coefficient information as in Refs.[11, 14], and similar results could be obtained.

III. SIMULATION AND DISCUSSION

All the simulation results shown here are obtained after discarding the first 10^5 time steps (as transient time) and then
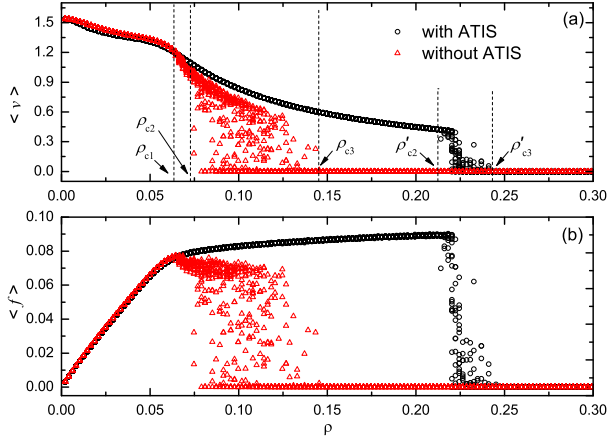
FIG. 2: (Color online) The average velocity (a) and the average flux (b) of each run in the system without and with ATIS.

averaging over the next 10^4 time steps. The system size is $N \times N = 24 \times 24$, and the length of street is $L = 100$. In NS rules, the maximum velocity of vehicles is $v_{max} = 3$ and the probability of random braking is $p = 0.1$. The traffic light period parameter T is set to 20, unless otherwise mentioned.

A. Clockwise strategy

This subsection studies the situation that the green lights change simultaneously in a clockwise manner at all intersections. Fig.2 shows the average velocity and the average flux of each run in system without ATIS. The average flux $\langle f \rangle$ of the system is defined as the average flux of all roads in the system. One can see that three different states could be identified.

When the density is smaller than a critical density $\rho_{c1} \approx 0.063$, the system is in moving state, in which the average flux almost linearly increases with the increase of density. Fig.3(a) displays typical pattern of the distribution of vehicles in the moving state, employing a grid coarsening method in which vehicle density of lattice S is defined as the average vehicle density of the four roads around the lattice (see Fig.1(a)). It can be seen that distribution of vehicles is heterogeneous, more vehicles accumulate in the center. The inner rings are circles. The outer rings gradually become squares.

When the system density is in the range $\rho_{c1} < \rho < \rho_{c2} \approx 0.073$, the system is in a saturation state, in which the flow rate almost saturates (it only slightly changes with the increase of density). When the system density exceeds a third critical density $\rho_{c3} \approx 0.145$, the local density in the center area becomes so large that a local deadlock will be induced. The local deadlock exhibits loop structure (see Fig.1(b)) in which all the desired outgoing streets of leading vehicles on the ingoing streets are in jam. This deadlock spreads to the system and causes global deadlock, in which no vehicle can move.

When the density is in the intermediate range $\rho_{c2} < \rho < \rho_{c3}$, the system is metastable: it could either evolve into global deadlock state or be in a saturation state. The scattered data indicate that the system is transiting from saturation state to

global deadlock state in the data collection period (from time step 10^5 to 1.1×10^5). Fig.3(c) shows typical pattern of the distribution of vehicles in the saturation state at $\rho = 0.1$. One can see that the local density ρ_l in the center area could reach very large value ($\rho_l \approx 0.7$). Moreover, the shape of the outer rings gradually changes from squares into diamonds.

Now we study the effect of ATIS. Fig.2 also shows the average velocity and the average flux of each run in system with ATIS. Similarly, the three states mentioned above are observed. Nevertheless, the system becomes much more stable with the help of ATIS: The saturation state is stable in the density range $\rho_{c1} < \rho < \rho'_{c2} \approx 0.212$. Moreover, the average velocity and the flow rate of the saturation state are also enhanced with the help of ATIS. The global deadlock state occurs when the density $\rho > \rho'_{c3} \approx 0.243$. The system is metastable in the density range $\rho'_{c2} < \rho < \rho'_{c3}$, which is much narrower than that in system without ATIS.

Fig.3(b) shows the distribution of vehicles in the moving state in the system with ATIS. Since the ATIS could help drivers to avoid center area, the accumulation of vehicles in the center is suppressed: the local density in the center area becomes much smaller (c.f.Fig.3(a)). Moreover, different from the system without ATIS, the inner ring exhibits diamond instead of circle.

Fig.3(d) shows the distribution of vehicles in the saturation state at $\rho = 0.1$. A four-angle-star structure begins to appear. More interestingly, with the further increase of density, a windmill-like high density structure appears (Fig.3(e)). The structure exhibits remarkable deviation from the diagonal lines. Due to symmetry, if the green lights change simultaneously in an anti-clockwise manner, the structure will deviate from the diagonal lines in an opposite way (see Fig.3(f)). Appearance of the structure might be related to the betweenness distribution because betweenness reaches the maximum on the diagonal lines. However, more efforts are needed in future work to explore the exact origin of the four-angle-star structure and the windmill-like structure.

Next we investigate the influence of traffic light period. Fig.4 compares the average velocity of the system at different values of T, in which the curves are obtained by averaging over many runs. At small densities, the average velocity decreases with the increase of T, with oscillations appearing (see Fig.5). The oscillations could be analyzed by considering a minimal network with one single intersection, as explained in details in Ref.[15]. Specifically, when traffic light period parameter $T > T_f = L/(v_{max} - p) = 34.5$, the local minimum appears at $T \approx nT_f$ ($n = 1, 2, \cdots$). When $T < T_f$, the local maximum appears at $T \approx T_f/n$ ($n = 2, 3, \cdots$).

We also need to point out in the moving state, the average velocity in the system with ATIS is smaller than that without ATIS. The difference is very remarkable when T is large (Fig.5), but it gradually shrinks with the decrease of T. The difference is due to the different distributions of vehicles (c.f.Figs.3(a) and (b)). While ATIS decreases the value of local density in the center area, the local density in the outer rings increases. While the former effect tends to increase average velocity, the latter one tends to decrease average velocity. In our case (Fig.5), the latter effect prevails over the for-
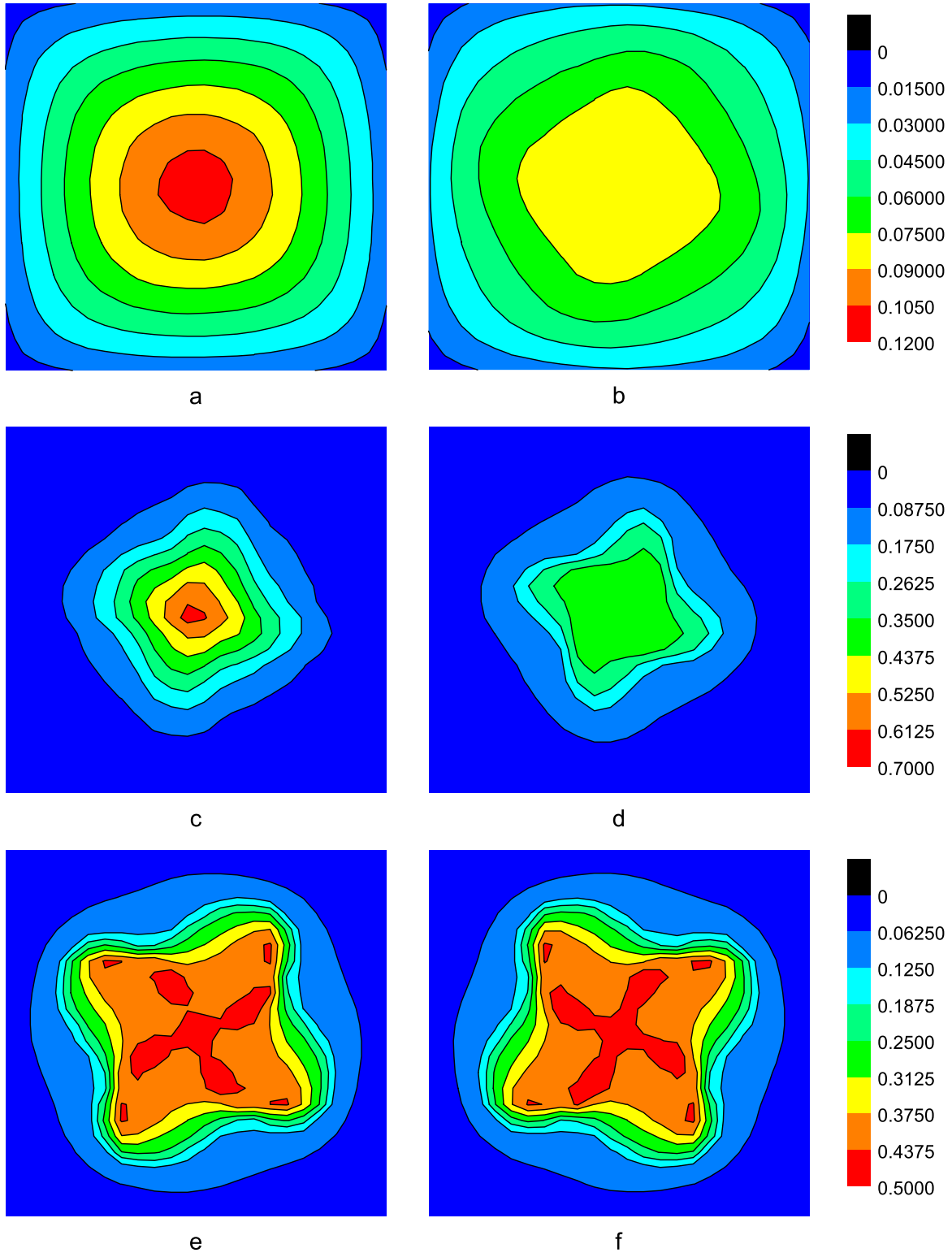
FIG. 3: (Color online) Typical patterns of the distribution of vehicles. (a), (b) $\rho = 0.05$; (c), (d) $\rho = 0.1$; (e), (f) $\rho = 0.15$. In (a) and (c), system has no ATIS; In (b) and (d-f), system has ATIS. In (a-e), the traffic lights are switched in clockwise manner; in (f), the traffic lights are switched in anti-clockwise manner. Note the scale of the colors is different at different rows.
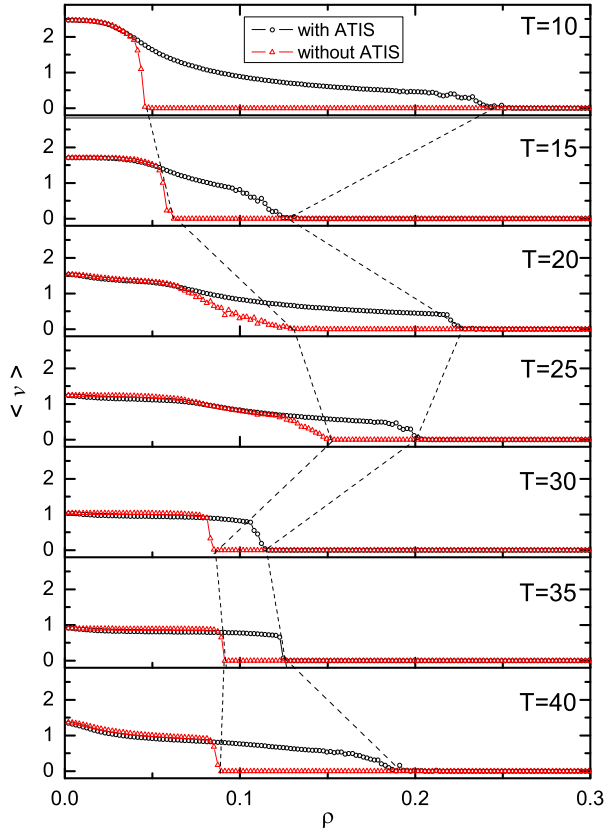
FIG. 4: (Color online) The average velocity of the system at different values of T. The dashed lines are guide for eyes, which roughly connect the transition regime from saturation state to global deadlock state.

mer one. Thus, the average velocity decreases in system with ATIS.

Finally, we focus on the transition regime from saturation state to global deadlock state. It can be seen that the transition regime heavily depends on T in both systems, and the transition always occurs later with the help of ATIS. Moreover, the transition is more sensitive to T in the system with ATIS: In the range $10 \leq T \leq 40$, there exist three extremes (The transition occurs later at the extremum than in the vicinity range) at $T \approx 10, 20, 40$ in the system with ATIS while there exists only one extremum at $T \approx 25$ in the system without ATIS. We point out that the average velocity also reaches maximum at $T \approx 10, 20, 40$ in the moving state (Fig.5). Whether there exists an underlying relationship for the coincidence or not needs to be explored in future work.

B. 8-like strategy

In this subsection, we briefly study a different synchronous traffic light strategy, in which the green lights are given alternatively for the horizontal streets and the vertical streets. Denote the traffic lights as 1, 2, 3, 4 in the clockwise manner, the lights are switched in the order 1-3-2-4-1 or 1-3-4-2-1 in
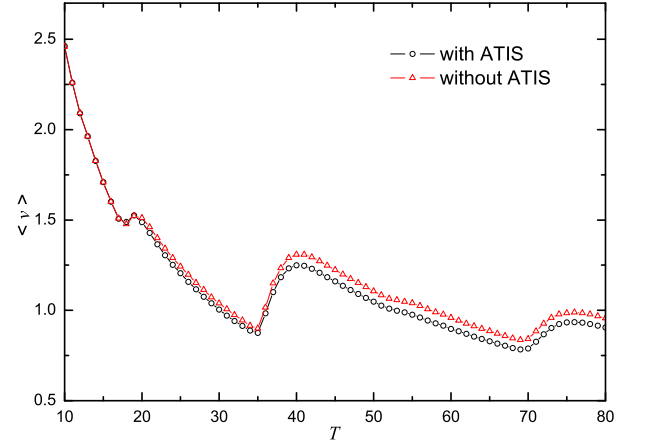

FIG. 5: (Color online) The average velocity of the system at different values of T. The system density $\rho = 0.01$.

this strategy. The former is named 8-like strategy and the latter is named reverse 8-like strategy, since the switching way looks like a "8".

Our simulation shows that similar results could be obtained as in previous subsection: the moving state, the saturation state and the global deadlock could be observed. Nevertheless, in the system with ATIS, the distribution of vehicles is much different. Fig.6 shows that in the moving state and the saturation state, the symmetry has been broken under the 8-like strategy. Instead of four-angle-star structure and windmill-like structure, the distribution exhibits Y-like structure (density between 0.525 and 0.6), T-shirt-like structure (density between 0.45 and 0.525) and triangle structure from inner to outer ring. We also point out that if the traffic lights are switched in the order 3-1-2-4-3 or 3-1-4-2-3, the vehicle distribution will exhibit symmetry with respect to that shown in Fig.6.

On the other hand, in the system without ATIS, the distribution of vehicles still remains symmetric as that under clockwise strategy (not shown here). Therefore, although the exact formation mechanism of various structures of the vehicle distribution is still unclear, it at least depends on (i) traffic light switching strategy, (ii) whether the ATIS is provided or not, and (iii) the topology of the network.

The traffic light switching strategy also has nontrivial impact on the transition from saturation state to global deadlock. Fig.7 compares the average velocity of the 8-like strategy with that of the clockwise strategy at four typical values of T. In system with ATIS, 8-like strategy performs worse than clockwise strategy except at $T = 30$. On the other hand, in system without ATIS, 8-like strategy never performs worse than clockwise strategy.

IV. CONCLUSIONS

In summary, a cellular automata model is proposed to study the traffic flow in Manhattan-like urban system. We have considered the origin-destination trips and traffic lights in this
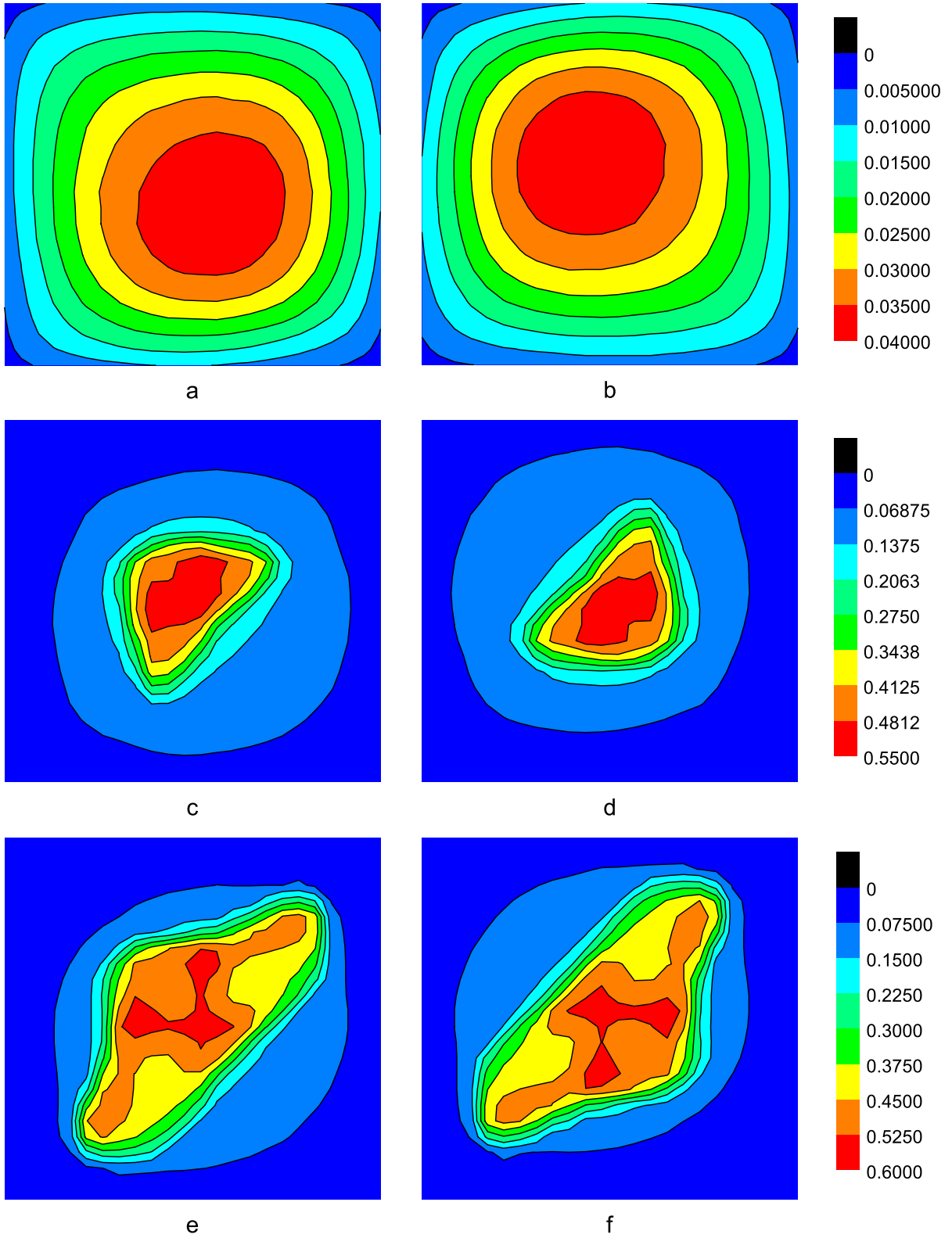
FIG. 6: (Color online) Typical patterns of the distribution of vehicles in system with ATIS. (a), (b) $\rho = 0.02$; (c), (d) $\rho = 0.1$; (e), (f) $\rho = 0.15$. In (a,c,e), 8-like strategy is adopted; in (b,d,f), reverse 8-like strategy is adopted. Note the scale of the colors is different at different rows.
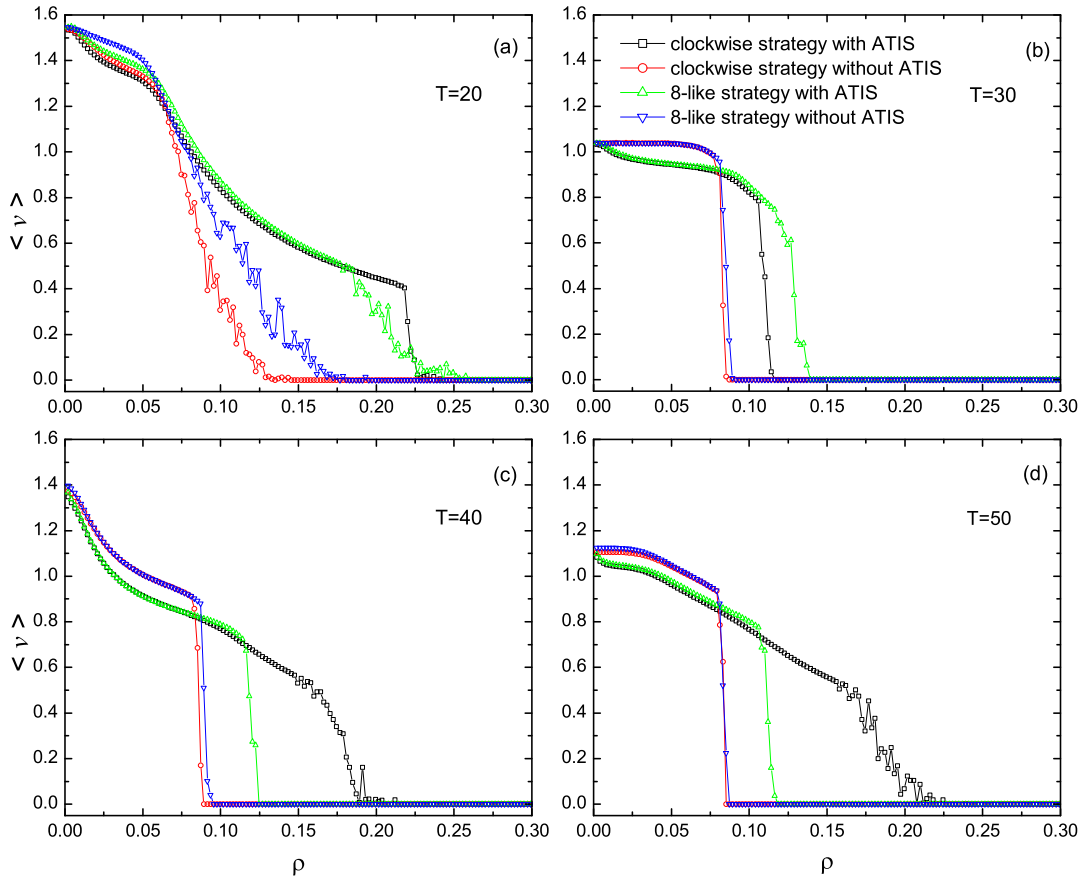
FIG. 7: (Color online) Comparison of the average velocity of two different strategies at four typical values of *T*.

model. It is found that the system could exhibit three different states, i.e., the moving state in which the flow rate increases with the density almost linearly, the saturation state in which the flow rate only slightly changes with density, and the global deadlock state in which no vehicle can move. With a grid coarsening method, we explore the distribution of vehicles in the moving state and the saturation state, which shows qualitatively different structure. A metastability of the system is observed in the transition from saturation state to global deadlock state. The influence of ATIS, the traffic light period, and the traffic light switch strategy on the system have been investigated.

The simple model in this paper needs to be extended in several directions in future work. (i) The routing strategy: optimal strategy needs to be designed to further enhance the system stability as well as the flow rate in the saturation state; (ii)

Traffic light strategy: other strategies (e.g., green wave strategy, adaptive traffic light strategy) need to be investigated or designed [16–18]; (iii) System structure: more realistic road network and multilane road sections need to be considered; (iv) More realistic origin-destination data need to be collected.

Acknowledgments

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[19] Sometimes there is only one shortest path, e.g., the origin and the destination are on same row or column.